

Fig. 4. Distribution of total effective cost for the phase shifter and driver in the text example. The effective cost of the phase-shifter imperfections amounts to 37 percent of the cost of the phase shifter and driver for this case.

numbers allows a satisfactory design to be achieved with few iterations.

Many times, the phased array designer is faced with a problem of selecting between diode and ferrite phase shifters having different initial costs and insertion losses. The effective system cost as outlined here provides a quantitative way of measuring the performance factors in terms of dollars as a common denominator. Similar approaches may be applied to the case where the aperture

is constrained so that a nonoptimum power  $\times$  aperture trade is dictated. This will generally result in a higher power per element and thus place a much higher premium on the insertion loss and other loss related performance parameters.

Evaluation of the effective cost of phase-shifter loss factors based on differential transmitter cost is valid for the optimum design case where it costs just as much to improve performance by a decibel in either transmitter or array. In the nonoptimum design case where the aperture is limited, it would be less costly to make up for a loss by an increase of array size, but that is not possible, so it is still valid to use the differential transmitter cost. If a non-optimum design is forced in which the transmitter is too small, it would be appropriate to evaluate phase-shifter losses in terms of differential array costs.

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# Comments on the Design and Manufacture of Dual-Mode Reciprocal Latching Ferrite Phase Shifters

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(Invited Paper)

**Abstract**—The design principles for dual-mode reciprocal latching ferrite phase shifters are relatively well understood at present. Discussions of a few selected topics not previously studied are presented in this paper. A tradeoff analysis is carried out for  $X$ -band units to show the interrelation between phase-shifter weight and insertion loss. An interesting consequence of this analysis is the theoretical prediction of an optimum range of values for the saturation moment of the ferrite material. Switching energy in the presence of shorted-

turn damping is also analyzed and related to the geometry and hysteresis loss of the ferrite material. Finally, a discussion of manufacturing considerations and unit cost at high rates of production is carried out. The major conclusion is that unit cost levels approaching \$10.00 are possible for a production run sufficiently large to justify the substantial cost of engineering and tooling for high rates of manufacture.

## INTRODUCTION

PHASED-ARRAY antenna needs have stimulated a wide variety of exploratory ferrite phase-shifter work in recent years. The main objectives of this work have

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been directed toward one or more of the goals of a) reduced cost, b) improved performance, or c) improved configuration. One of the recent successes has been in the area of reciprocal latching phase shifters, where a really practical design has become available. The first hint of solution to this long-standing problem was published several years ago [1], with much improved versions reported more recently [2], [3]. These designs all use a variable longitudinal field in a dual-mode guide, with opposite senses of circular polarization for transmitted and received energy. The key element that has made this approach practical has been the extremely efficient nonreciprocal circular polarizer used at the ends of the variable-field section. These polarizers are achieved in the same ferrite rod simply by externally applying a transverse quadrupole biasing field to the rod. Thus a single ferrite rod is used for two polarizers and a variable-phase section. The latter is fitted with external yokes of ferrite for latching the variable longitudinal field. Because the rod is long and slender, this arrangement yields remanent field values almost at the level of a solid toroid.

The potential cost advantages to this approach are clear. First of all, there is no solid toroid, and the ferrite rod can be easily and cheaply machined to very accurate tolerances. As far as performance is concerned, the arrangement makes such good utilization of the ferrite that loss levels comparable to the nonreciprocal toroid-type phase shifters are obtained. One significant drawback is the fact that the external yoke requires switching through a thin metal waveguide wall. As a result, eddy currents are induced in the wall, and switching energy is increased substantially at high speeds. At the current state of the art, switching speeds are about an order of magnitude slower than the nonreciprocal toroid. This may not be a problem for some system applications, such as radar antennas, since the array may not need to be reset quickly between the transmit and receive configurations.

The sections following summarize the results of studies carried out on the design of an X-band ferrite phase shifter intended for possible use in a phased-array antenna. The constraints of the particular application are such that low weight, small size, and low cost are of prime concern. As a result, the computations were focused on ferrite materials of the lithium-titanium family. These materials do not use expensive rare-earth elements as constituents, in contrast with YIG-type materials, and have much better temperature stability than magnesium-manganese (MgMn) ferrites. Also, the relative dielectric constant of lithium-type ferrites is higher than either YIG or MgMn ferrite, which aids in size and weight reduction.

### PHASE-SHIFTER DESIGN PRINCIPLES

The simplest dual-mode phase shifter uses a completely filled geometry in which the surface of the ferrite rod is metallized to form a waveguide wall. The amount of phase shift available from the variable-field axially magnetized section depends on the saturation moment, remanence

ratio, operating frequency, and cross-sectional dimensions of the guide. Fig. 1 reproduces a family of curves of normalized phase shift for the infinite-medium case [2]. In these curves, the normalizing insertion phase value  $\phi_0$  is the insertion phase of a medium having the same dielectric constant as the ferrite, but unity relative permeability. Computations for these curves were made taking into account the fact that the initial microwave relative permeability of the medium is generally not unity. The phase shift  $\Delta\phi_g$  for a rod of finite cross section can be calculated directly using standard perturbational methods, or estimated with reasonably good accuracy from the infinite-medium curves using the approximation

$$\Delta\phi_g \simeq \Delta\phi\lambda_0/\lambda_g$$

where  $\lambda_0$  and  $\lambda_g$  are, respectively, the infinite medium and guide wavelengths in the equivalent dielectric medium ( $\mu_r = 1$ ,  $\epsilon = \epsilon_0\epsilon_r$ ), and where  $\Delta\phi$  is the infinite-medium phase shift, as above.

Limitations on the frequency bandwidth arise from impedance-matching problems and from the dispersion of the differential phase shift of the nonreciprocal circular polarizers. Impedance matching of the fully filled ferrite guide to a rectangular air-filled guide has been done by using a section of ceramic dielectric rod that forms an extension of the ferrite rod. This ceramic rod typically projects through the end wall into the rectangular guide for some distance. By using a two-section stepped rod in the rectangular guide, and with some attention to discontinuity immittances, it is possible to attain good match bandwidths on the order of 30 percent of center frequency. Fig. 2 shows a swept trace of the return loss characteristic of a dual-mode phase shifter with two-section matching in which the phase states were scanned rapidly and frequency scanned slowly to display all values obtained. The nonreciprocal circular polarizers in a fully filled guide have an inherent differential phase-shift characteristic that varies slightly faster than  $1/f$ . Excursions from the nominal band center frequency cause deviation from the  $90^\circ$  differential phase-shift point, and the phase shifter suffers from an alignment error. The principal effect of this error is to cause increases in the insertion loss and insertion loss modulation levels.

The insertion loss depends on the dielectric and magnetic loss tangents of the material, the wall conductivity, and the alignment of the device. The base dielectric loss is typically negligible compared with the magnetic loss and wall loss. In a well-designed low-power structure, the conductive loss will usually exceed the magnetic loss by a factor of 2 or 3. When high-power materials are used, the magnetic loss at a given saturation moment increases and may begin to dominate. For this reason, it is preferable to use a lower saturation material and allow the length to increase somewhat so that the magnetic loss can be significantly reduced at the expense of a slight increase in wall loss. The insertion loss typically varies up to about 0.3 dB above the base loss with phase changes

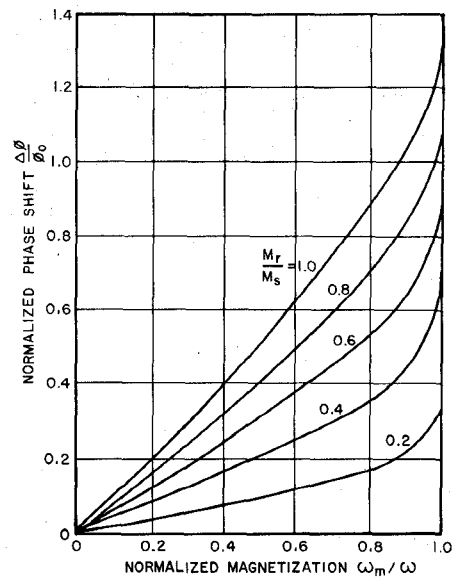


Fig. 1. Normalized phase shift for a circularly polarized plane wave in a remanently magnetized infinite ferrite medium.

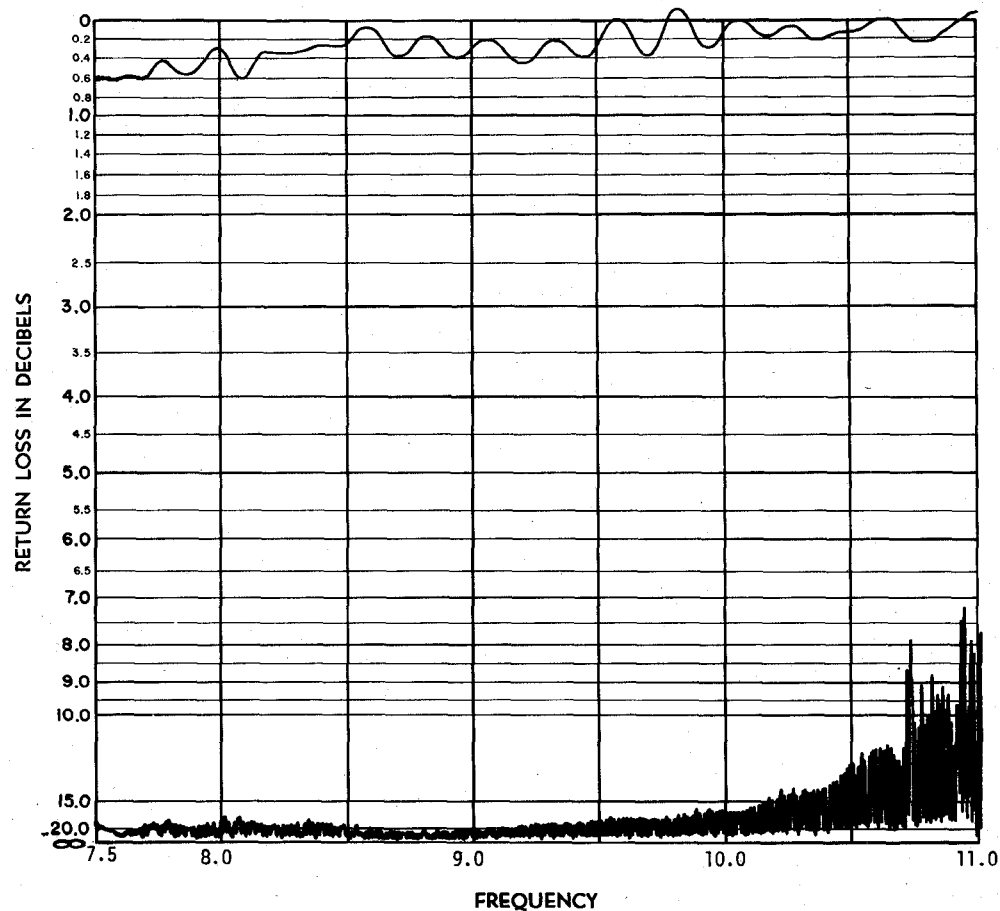


Fig. 2. Return loss trace for a dual-mode phase shifter with broad-band matching.

because of alignment tolerances, variable mismatch, and changes in magnetic loss. Fig. 3 shows a typical insertion loss characteristic with phase scanning to display the complete loss envelope.

Temperature stability and unit-to-unit matching of

insertion phase are significant problem areas that must be taken into account. Better stability of insertion phase with temperature is obtained by using the longest phase state as the reset point. Experience indicates that an insertion phase match of about  $\pm 15^\circ$  is obtainable for a

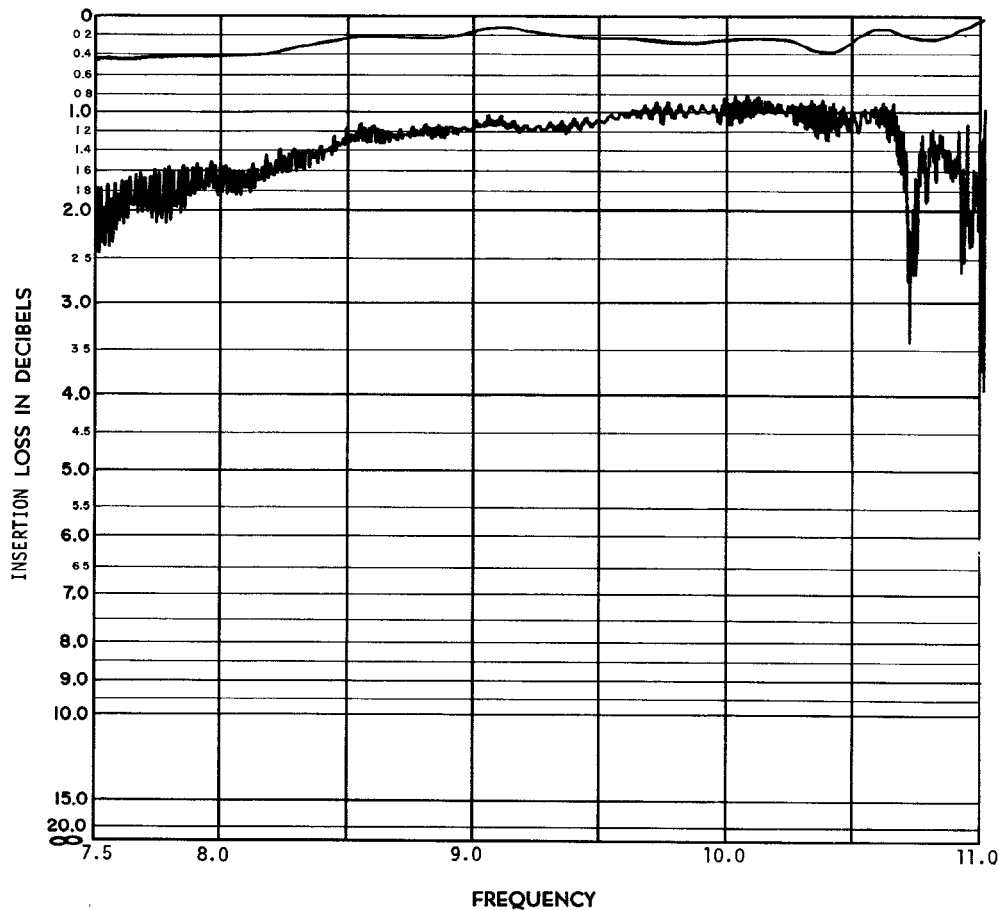


Fig. 3. Insertion loss trace for same phase shifter as in Fig. 2.

quantity of units made from a single batch of ferrite. This remaining spread can be corrected electronically by a variable "preset" that establishes a uniform zero state slightly up the hysteresis loop for each phase shifter. To accommodate for temperature variation and insertion phase matching, it is desirable to make units with substantially more major-loop phase shift than the amount actually needed in the system application.

#### TRADEOFF ANALYSIS

In two-dimensional phased-array antennas, the parameters of low weight, small size, and low cost are generally of prime concern. Extensive computations have been carried out for the dual-mode phaser to define the trade-offs involved between material selection, weight, and insertion loss. In these calculations, lithium-titanium ferrites in the saturation moment range of 400–3400 G were considered as potential candidates for an X-band phaser. The assumptions made were that a remanence ratio of 0.75 and dielectric loss tangent of  $4 \times 10^{-4}$  were independent of saturation moment and that the demagnetized magnetic loss characteristic at X band was in the same proportion of  $(\omega_m/\omega)^2$  for each composition. An approximate value for the magnetic loss coefficient was supplied on the basis of experimental results on a phaser assembly.

The analytical model for the magnetic behavior of the ferrite medium was based on work recently reported re-

garding the characteristics of partially magnetized ferrite [4]. The microwave initial permeability  $\mu_i$  is expressed as

$$\mu_i = \frac{1}{3} + \frac{2}{3} \left[ 1 - \left( \frac{\omega_m}{\omega} \right)^2 \right]^{1/2} \quad (1)$$

where  $\omega_m = \gamma 4\pi M_s$ ,  $\gamma$  is the gyromagnetic ratio,  $M_s$  is the saturation moment, and  $\omega$  is the radian frequency. The elements of the transverse Polder tensor are taken as functions of the operating magnetic moment  $M_r$  as follows:

$$\mu_a = \mu_i + (1 - \mu_i) \frac{\tanh(1.25 M_r / M_s)}{\tanh 1.25} \quad (2)$$

and

$$\kappa = \frac{M_r \omega_m}{M_s \omega} \quad (3)$$

These relations were used in standard perturbational formulas to compute the anticipated phase difference between the cases where  $M_r$  had the maximum positive and negative remanent value.

Ordinary perturbational formulas were also used to calculate the expected insertion loss, which is considered to be the result of conductive wall loss, dielectric loss, and magnetic loss of the ferrite. As indicated previously, the loss model for magnetic loss was taken as

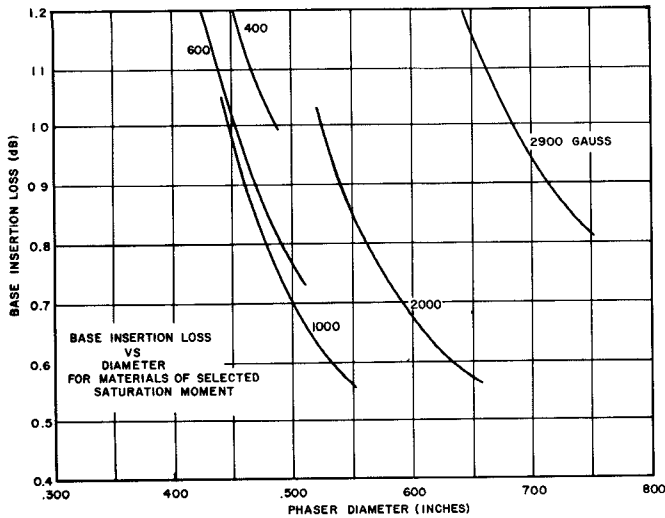


Fig. 4. Computed loss dependence on diameter for phase shifters using lithium-titanium ferrite.

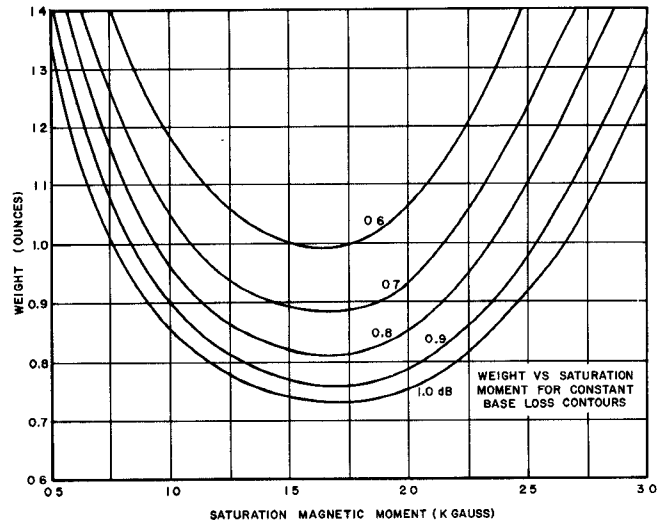


Fig. 6. Computed constant-loss contours of weight dependence on saturation moment.

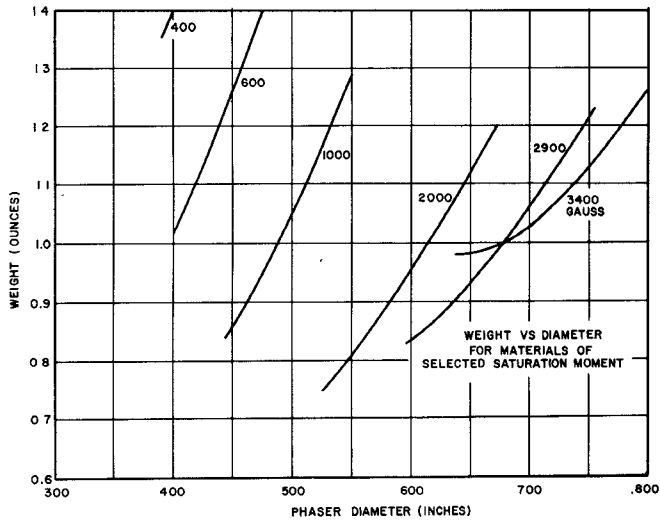


Fig. 5. Computed weight dependence on diameter for phase shifters using lithium-titanium ferrite.

$$\mu'' = A \left( \frac{\omega_m}{\omega} \right)^2. \quad (4)$$

The parameters computed were the length of rod required to achieve 500° latching phase shift at midband, dimensions of polarizers and yokes, phase-shift dispersion, weight, and loss breakdown by conductive, dielectric, and magnetic loss over the frequency band. The same high-saturation yoke material was assumed for all designs.

The operating frequency was assumed to be in the range of 9.2–10.2 GHz. Fig. 4 shows a plot of the expected insertion loss as a function of diameter of the entire phaser assembly, for materials of various saturation moment. Fig. 5 plots the expected element weight as a function of phaser diameter. Using these two families of curves, a set of contours of element weight versus ferrite saturation moment have been plotted in Fig. 6. These curves clearly show that an optimum choice of material saturation moment exists that minimizes the element

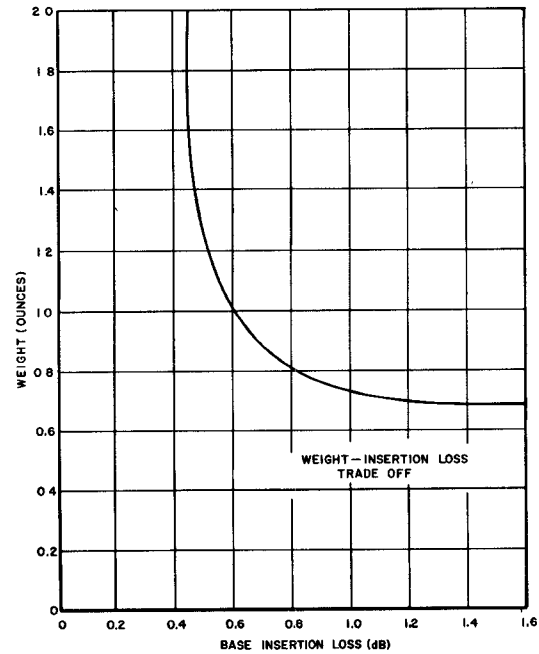


Fig. 7. Computed interdependence between minimum weight and insertion loss.

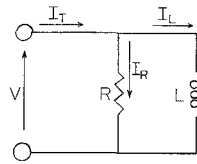
weight for a given insertion loss level. By choosing the best material at each loss level, it is possible to generate a curve that displays the optimum weight versus base loss relationship. This curve, shown in Fig. 7, has the shape of a rectangular hyperbola. The significant features of this curve are that it predicts an absolute minimum of insertion loss regardless of element weight, and an absolute minimum of element weight even if insertion loss is allowed to be arbitrarily large. Also, the region of reasonable compromise between weight and base loss is fairly well defined by this curve.

For very large-scale production, it is likely that the cost of materials will be a significant portion of the total phaser cost. A design that minimizes weight should also

result in the least volume of ferrite required and, consequently, tend to minimize cost. There are, of course, many other factors that can influence the design of a phaser for any specific system, such as the power handling requirements, cross-sectional size constraints, length constraints, and so on. Also, the state of materials development continues to advance, with potential impact on future designs.

### PHASE-SHIFTER SWITCHING CONSIDERATIONS

Switching considerations are another important aspect of phase-shifter design. In the usual case, the phase shifter is driven from a voltage source with very low dynamic impedance. The amount of flux change is metered by controlling the duration of applied voltage pulses, with the current allowed to conform to whatever waveform is necessary. The current level is determined by three factors: 1) hysteresis effects, 2) shorted-turn damping, and 3) the driving inductance. It is only the latter that relates to the desired flux change. The shorted-turn damping effect can be represented in equivalent-circuit form by a linear resistance shunting the inductance of the driven ferrite configuration:



The current through the equivalent inductance can be expressed as

$$I_L = I_c + \frac{N}{L} \phi$$

where  $I_c$  is the "coercive current," related to the coercive force  $H_c$  by

$$I_c = \frac{kH_c l}{N}$$

In the preceding expressions,  $N$  is the number of turns of the driving coil,  $\phi$  is the total magnetic flux linking the coil, and  $k$  is a proportionality constant to account for the system of units used. The current  $I_R$  through the equivalent shorted-turn resistance is directly related to the voltage  $V$  developed across the inductance:

$$I_R = \frac{V}{R} = \frac{N}{R} \frac{d\phi}{dt}$$

Thus the total current  $I_T$  will be

$$I_T = I_L + I_R = \frac{kH_c l}{N} + \frac{N\phi}{L} + \frac{N}{R} \frac{d\phi}{dt}$$

The total energy used in switching from one saturated state to the other will now be approximately calculated.

Assume that a constant voltage  $V$  is impressed across the coil for a time  $\tau$  to switch flux at a constant rate from  $-\phi_{\max}$  to  $+\phi_{\max}$ .

Then for  $0 \leq t \leq \tau$

$$\frac{d\phi}{dt} = 2 \frac{\phi_{\max}}{\tau}$$

$$\phi(t) = \phi_{\max} \left( 2 \frac{t}{\tau} - 1 \right)$$

and consequently

$$i_T(t) = \frac{kH_c l}{N} + \frac{N\phi_{\max}}{L} \left( 2 \frac{t}{\tau} - 1 \right) + \frac{2N}{R} \frac{\phi_{\max}}{\tau}$$

The switching energy will be

$$W = \int_0^\tau v i dt = V \int_0^\tau i_T(t) dt$$

$$W = V \left[ \frac{kH_c l \tau}{N} + \frac{2N\phi_{\max}}{R} \right]$$

and since  $V = 2N\phi_{\max}/\tau$ ,

$$W = 2kH_c l \phi_{\max} + \frac{4N^2 \phi_{\max}^2}{R\tau}$$

Now substitute

$$L = N^2 \frac{\phi_{\max}}{k(H_{\max} - H_c)l}$$

and get

$$W = 2kH_c l \phi_{\max} + 4 \frac{Lk}{R\tau} (H_{\max} - H_c) l \phi_{\max}$$

The quantity  $L/R$  is the dynamic shorted-turn damping time constant  $\tau$ , i.e., the time constant in effect when driving up the steep side of the hysteresis loop. Next, note that by defining

$$\phi_{\max} = B_{\max} A$$

where  $A$  is an effective cross-sectional area, the term  $2kH_c l \phi_{\max}$  is recognized as the volume integral of the hysteresis energy for half the hysteresis loop. This term will be defined as  $W_H$ , with the conclusion

$$W = W_H \left[ 1 + 2 \frac{\tau}{\tau} \left( \frac{H_{\max}}{H_c} - 1 \right) \right]$$

This equation expresses the driving energy in the presence of shorted-turn damping as a function of the hysteresis energy.

The time constant  $\tau$  is basically related to the geometry of the switched assembly. An approximate calculation of  $\tau$  reveals the essential parameters that govern its magnitude. For a yoke length  $l$  on a rod of diameter  $D$ , metalized to a thickness  $s$  with a material of resistivity  $\rho$ , the

shorted-turn resistance on a one-turn basis will be

$$R = \frac{\rho\pi D}{ls}.$$

Similarly, the inductance on a one-turn basis is given approximately by

$$L = \frac{\phi}{I} = \frac{6.452 \times 10^{-8} B_{\max} \pi D^2 / 4}{4.04 (H_{\max} - H_c) l}$$

$$L = 4.0 \times 10^{-9} \frac{B_{\max}}{H_c l} \frac{\pi D^2}{[(H_{\max}/H_c) - 1]}.$$

Forming the ratio  $\tau = L/R$ ,

$$\tau = 4.0 \times 10^{-9} \frac{B_{\max} D s}{H_c [(H_{\max}/H_c) - 1] \rho}.$$

Now substitute for  $\tau$  in the energy relation given previously:

$$W = W_H \left[ 1 + \left( \frac{8}{\tau} \right) \left( \frac{B_{\max}}{H_c} \right) \frac{Ds}{\rho} \times 10^{-9} \right].$$

The resistivity of copper at standard temperature is  $6.79 \times 10^{-7} \Omega \cdot \text{in.}$  If this value is used as a normalizing parameter,

$$W = W_H \left[ 1 + \left( \frac{11.8}{\tau} \right) \left( \frac{B_{\max}}{H_c} \right) \left( \frac{Ds}{\hat{\rho}} \right) \times 10^{-9} \right]$$

where

- $\tau$  remanence-to-remanence switching time in milliseconds;
- $B_{\max}$  remanent rod flux density in kilogauss;
- $H_c$  rod coercive force in oersteds;
- $D$  rod diameter in inches;
- $s$  metallized film thickness in mils;
- $\hat{\rho}$  metal film resistivity relative to copper.

The form of this equation lends itself to the defining of a reference time  $\tau_0$ , such that

$$W = W_H \left[ 1 + \frac{\tau_0}{\tau} \right]$$

where

$$\tau_0 = 11.8 \left( \frac{B_{\max}}{H_c} \right) \left( \frac{Ds}{\hat{\rho}} \right) \text{ ms.}$$

A sample calculation at  $X$  band will be made, based on the following assumed parameters:

$$\begin{aligned} B_{\max} &= 1.5 \text{ kG} \\ H_c &= 1.25 \text{ Oe} \\ D &= 0.3 \text{ in} \\ s &= 0.2 \text{ mil} \\ \rho &= 5. \end{aligned}$$

Then

$$\tau_0 = 11.8 \left( \frac{1.5}{1.25} \right) \left( \frac{0.3 \times 0.2}{5} \right) = 0.17 \text{ ms.}$$

The switching time available for the reset-set cycle is 0.05 ms; assuming the entire loop is traversed each cycle, the effective value of  $\tau$  can be conservatively taken equal to  $0.1\tau_0$ , and the energy will be on the order of 11 times the hysteresis energy. Now

$$W_H = 2.09 B_{\max} H_c \Lambda \times 10^3 \mu\text{J}$$

where  $\Lambda$  is the volume of switched rod in cubic inches. This expression has a factor of 2 incorporated in it to account for the yoke material, which is assumed to have hysteresis loss equal to that of the rod. In the typical case considered previously, a value of  $\Lambda = 0.14 \text{ in}^3$  is roughly correct, so that

$$W_H = 2.09 \times 1.5 \times 1.25 \times 0.14 \times 10^3 = 550 \mu\text{J.}$$

Then for  $\tau = 50 \mu\text{s}$

$$W = 11W_H = 6050 \mu\text{J.}$$

This value is in good agreement with experimentally observed results. A reduction of the time constant  $\tau_0$  by about a factor of 5 is possible by incorporating a longitudinal slot in the ferrite rod metallized surface. This slot must be covered by a capacitive overlay to prevent RF leakage. Using this type of construction, the value of  $\tau_0$  can be reduced from 0.17 ms to around  $34 \mu\text{s}$ . This means that the switching energy will be reduced from around  $11W_H$  down to a level roughly equal to  $3W_H$ . Thus, with the capacitive overlay wall metallization,

$$W = 3W_H = 1650 \mu\text{J.}$$

Although the capacitive overlay approach is more costly to implement, the significant reduction in switching energy may make it worth serious consideration in some systems.

## MANUFACTURING CONSIDERATIONS

One of the major factors limiting widespread application of two-axis scanned phased-array antennas has been the high cost of the large number of phase control elements. The dual-mode latching reciprocal ferrite phase shifter has not only the characteristics required to provide high performance, but also has a structure compatible with low unit cost in production. Achieving this low unit cost will require a significant amount of manufacturing engineering to establish optimum materials and processes, as well as careful planning of work flow. Some basic assumptions must be made about necessary prerequisites to low-cost production.

1) Electrical and mechanical design must be compatible with low unit cost.

2) Performance specifications must be established at levels consistent with high-yield factors.

3) Adequate funding and lead time must be made available for high-volume production tooling.

4) Total quantity of parts to be produced must be consistent with high production rates over an extended period of time.

5) Data requirements must be minimal.

Development effort on the dual-mode phase shifter has continued at a moderate level over a period of roughly eight years. Although advances in materials technology can always impact established designs, it is hard to imagine major breakthroughs that would significantly simplify the electrical or mechanical design of the dual-mode phase shifter. As currently conceived, the unit already has design features that are compatible with low unit cost. Electrically, the RF structure is simply a fully filled ceramic waveguide, completely separated from the switching and control circuitry. The functions of linear-to-circular nonreciprocal polarizing and depolarizing, and of longitudinal field switching, are accomplished by fitting elements onto the ferrite rod *outside* the RF field region. These external elements are also not mechanically complicated, consisting mostly of quadrupole-field permanent magnets, a coil with a small number of turns, and a yoke to provide an external magnetic return path.

Fortunately, the performance requirements on elements of a two-axis scanned phased-array antenna are usually not severe with respect to phase accuracy. That is, a considerable amount of random error can be accommodated at the element level without serious degradation of array performance in most practical cases. This allows the performance specification for the individual phase shifters to be set at levels that are attainable in a statistical sense with high percentage yield in production. Suggested values for a lightweight X-band phase-shifter array element are

frequency band	10 percent;
insertion loss	1.0-dB average, 1.2-dB maximum limit over 80 percent of phase states, 1.5-dB absolute maximum;
VSWR	1.2:1 typical, 1.5:1 maximum;
phase-shift range	500° nominal;
phase set deviation from command angle	±5° median deviation, ±10° limit over 80 percent of phase states, ±20° absolute maximum;
switching time (set to set)	50-μs nominal cycle time;
weight	1.0-oz nominal.

With performance requirements at these levels, it is anticipated that a yield of at least 95 percent would be achieved in high-volume production.

A significant amount of manufacturing engineering and production tooling will be necessary to achieve the goal of low unit cost. The end result of this effort would be to have in place the machinery, fixtures, and special dies and tools needed for high-volume production. For example, extrusions may be developed for polarizer magnet parts

and possibly also for the ferrite rod itself. Modifications to production machinery, holding fixtures, and related tooling hardware will need to be developed and implemented to allow fabrication, assembly, and testing operations to proceed as rapidly as possible. Pilot-line proof testing of the production setup will also need to be carried out. It is roughly estimated that a period of two years will be required to carry out these tasks, at a cost of around  $1 \times 10^6$  dollars, for the phase shifter exclusive of driver electronics.

The phrase "high-volume production" has been used without any clear definition of its meaning. In carrying out cost estimates, a production rate of 1500 units/working day has been assumed. This rate is well within the capacity of existing production machinery and would require a labor force of only around 20 direct workers for assembly and test. This rate is judged to be a good compromise level; at much lower rates, the expense of tooling would not be justified because of significant overcapacity, and the unit cost would therefore rise appreciably. Moderately higher rates of production would most likely be achieved by duplication of facilities, with minimal savings of unit cost. At the assumed production rate, a run of nearly three years would be required to amortize the nonrecurring manufacturing engineering, production tooling, and start-up costs at the level of \$1.00/unit. Assuming that the phase shifters would be incorporated into an array containing 2000 elements, a quantity of only 500 antennas would be sufficient to justify a program at the level postulated. Using the estimated figures of Table I, the cost of phase shifters for each antenna would be around  $2.4 \times 10^4$  dollars, including tooling amortization. While this is a substantial sum, it is not likely to be the driving parameter in some system level decisions regarding the use of an electronically steerable phased-array antenna.

The cost estimate figures shown in Table I are based on current judgments regarding the labor and materials

TABLE I  
ESTIMATED PRODUCTION COST—X-BAND DUAL-MODE PHASE SHIFTERS

Item	Cost
Ferrite rod, centerless ground	\$ 2.60
Magnetic yoke parts	2.08
Magnetic polarizers	0.74
Ceramic impedance matching transformers	1.56
Miscellaneous materials (bushings, wire, epoxy, etc.)	0.43
Assembly labor	3.40
Test labor	0.17
Total	\$ 10.98



required to produce an existing phase-shifter design at high volume. These figures indicate the distribution of cost and include attrition, burden, and profit. The attrition rates assumed were 10 percent on metal parts, 30 percent on ceramic parts, and a 4-percent rejection rate of completed assemblies at the final inspection level. The costs of materials are based on current prices on actual vendor quotations. Fabrication and assembly use known and proven techniques. Consequently, the figures of Table I are considered to be conservative with regard to ultimate high-volume production cost of an X-band dual-mode phase shifter.

### CONCLUSIONS

A substantial body of knowledge exists about the design and manufacturing considerations for the dual-mode reciprocal latching ferrite phase shifter. Fundamental design techniques are well established for providing reasonable performance over moderately wide frequency bands. The weight-insertion loss tradeoff has been explored

analytically and found to predict an optimum choice of material for low RF power applications. Parameters affecting switching and, in particular, shorted-turn damping, have been identified and expressed in normalized form for convenience. Finally, the manufacturing problem has been worked in far greater detail than the scope of this paper allows, to the conclusion that lightweight X-band phase shifters can be produced at high-volume rates for unit cost levels on the order of \$10.00.

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# Computer Analysis of Microwave Propagation in a Ferrite Loaded Circular Waveguide—Optimization of Phase-Shifter Longitudinal Field Sections

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**Abstract**—A theoretical analysis of the microwave propagation in a circular TE<sub>11</sub> waveguide partially or completely loaded with an axially magnetized ferrite rod is presented. This study is based upon an exact analytical treatment of the Maxwell's equations, together with an original numerical method of solving transcendental equations with a complex variable. The introduction of the complex propagation constant  $\Gamma = \alpha + j\beta$ , taking in account the losses in the filling medium, had never been attempted because of the mathematical difficulties involved making essential the use of a large capacity computer. The developed program not only supplies all the propagation characteristics for a given structure but also enables us to optimize a phase-shift section in accordance with the user's requirements.

This study is a first step towards the theoretical optimization of two types of reciprocal phasers: the dual mode phaser (DMP) and

the polarization insensitive phaser (PIP), both widely used in array antenna systems. The computed results obtained for the basic section of such phasers operating at a central frequency of 9.5 GHz are given.

Obviously, this work is still incomplete since it does not include the optimization of all the components of a practical phase shifter, for example, the polarizers. Moreover, we have assumed the ferrite partially magnetized by a continuously variable bias field, although the PIP and the DMP are normally operated in a latching configuration; we presently complete our study according to these practical considerations.

### NOMENCLATURE

$$\beta_0^2 = \omega^2 \epsilon_0 \mu_0, \quad \lambda_0 = 2\pi/\beta_0, \quad \eta_0 = \left[ \frac{\mu_0}{\epsilon_0} \right]^{1/2}$$

$$k_d^2 = \beta_0^2 \epsilon_d \mu_d + \Gamma^2 \quad k_{d'}^2 = \beta_0^2 \epsilon_d \mu_d + \Gamma'^2$$

$$k_f^2 = \beta_0^2 \epsilon_f \mu_f + \Gamma^2 \quad k_{f'}^2 = \beta_0^2 \epsilon_f \mu_f + \Gamma'^2$$

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